Metasurface Waveguides Applied to Matched Feeds for Reflector Antennas

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***Abstract*—Waveguides with anisotropic surface impedance boundaries have been investigated for the purpose of matched feeds for offset reflectors. Matched feeds employ higher order waveguide modes to cancel out cross polarization introduced by the offset geometry. Since the higher order modes propagate at different speeds than the fundamental mode in conventional waveguides, it is challenging to meet phase relationship requirements over a large band. We have found that traditional corrugated waveguides are poorly suited for matched feed applications. However, other surfaces that satisfy the balanced hybrid condition, but have a small capacitive longitudinal reactance and large inductive azimuthal reactance show very promising properties: In a large band, HE_{11} and HE_{21} have similar propagation characteristics.**

***Index Terms*—metasurface, matched feed, offset reflector antenna.**

I. Introduction

Prime focus offset reflector antennas have many advantages, but suffer from high cross polarization [1]. It is a well known trade-off that a more compact system results in higher cross polarization. By using Bem's [2] analysis of the focal region field, Rudge and Adatia [3] proposed that the cross polarization be compensated by additional waveguide modes in the feed horn. For smooth walled circular horns, the TE_{21} mode can compensate cross polarization if excited in phase quadrature with the fundamental mode, TE_{11}, though other modes can also be used [4]. For corrugated horns, the corresponding compensating mode is HE_{21}.

The corrugated horn represents a special case of an anisotropic impedance boundary waveguide. Here we shall investigate the propagation of higher order hybrid modes in more general waveguides with anisotropic impedance walls. Such surfaces may be implemented by periodic structures on the horn/waveguide wall, i.e. metasurfaces.

Wu et al. and Scarborough et al. have successfully demonstrated horns with metasurface walls [5], [6], [7], [8] which exhibit very good performance over a wide band. However, their focus was not matched feeds, but a good co-polarized feed and thus did need analysis of modes with higher order azimuthal index.

By considering dispersion characteristics of the hybrid modes HE_{11} and HE_{21} in circular cylindrical waveguides with anisotropic boundary conditions, we shall investigate if there are metasurface waveguides suitable for matched feed applications. First, a brief theory of modal solutions in impedance waveguides will be given, following the approach of e.g. [9] and [10].

II. Theory

Consider a cylindrical waveguide with circular cross-section which extends along z. We assume solutions which are travelling waves in the positive z-direction with phase constant β, i.e. the field variation along z is $e^{-j\beta z}$, given a harmonic time factor of $e^{j\omega t}$. The longitudinal (z) components of the fields then satisfy the scalar Helmholtz equation. In a cylindrical coordinate system with its z-axis along the center of the waveguide, the longitudinal field components have solutions of the form

$$E_z(\rho,\phi,z) = E_0 J_m(k_c\rho) \begin{array}{c}\cos\\ \sin\end{array} m\phi \; e^{-j\beta z} \tag{1a}$$

$$H_z(\rho,\phi,z) = \pm E_0 \frac{\gamma}{Z_0} J_m(k_c\rho) \begin{array}{c}\sin\\ \cos\end{array} m\phi \; e^{-j\beta z}, \tag{1b}$$

from which it follows that the so-called *hybrid factor* [9] is

$$\gamma = Z_0 \frac{H_z(\rho,\phi,z)}{E_z(\rho,\phi-\frac{\pi}{2m},z)}, \tag{2}$$

which measures the ratio of longitudinal E- and H-fields for a solution. Z_0 is the free space impedance. The top and bottom choices of cos, sin, $+$, and $-$ correspond to two orthogonal solutions. The hybrid factor is zero and infinite for pure TM and TE modes, respectively. Any other value of γ is a hybrid mode: If the hybrid factor is positive, we denote the solution an HE hybrid mode, and if negative, an EH hybrid mode [9].

Given the simple z-variation of the field, the longitudinal components sufficiently characterize the field and the transverse components can be found from these:

$$\mathbf{E}_t = \frac{j}{k_c^2}\left(\omega\mu\hat{\mathbf{z}}\times\nabla_t H_z - \beta\nabla_t E_z\right) \tag{3a}$$

$$\mathbf{H}_t = \frac{j}{k_c^2}\left(\omega\epsilon\nabla_t E_z\times\hat{\mathbf{z}} - \beta\nabla_t H_z\right). \tag{3b}$$

We now impose boundary conditions in the form of normalized surface impedances at the wall of the waveguide

$$z_z = -\frac{E_z}{Z_0 H_\phi}, \quad z_\phi = \frac{E_\phi}{Z_0 H_z}. \tag{4}$$

The transverse field quantities are obtained from the longitudinal ones and evaluated on the boundary. The boundary conditions are then manipulated to obtain an equation which links the wavenumber, k, to the transverse wavenumber k_c — the characteristic equation:

$$jJ_m^2(k_c a)\left[z_z\frac{m^2}{(k_c a)^2}\left(\frac{k_c^2}{k^2}-1\right)-z_t\frac{k_c^2}{k^2}\right] + jJ_m'^2(k_c a)z_z - J_m(k_c a)J_m'(k_c a)\frac{k_c}{k}\left[z_z z_t+1\right]=0. \tag{5}$$

The characteristic equation is solved numerically for each k. For a given k, there may be several k_c which satisfy the equation, corresponding to different modal solutions.

III. Dispersion Analysis

Using the simple theory outlined in the previous section, we can investigate the modal solution of waveguides with different anisotropic wall impedances. A convenient way to analyze the modes is by dispersion diagrams. They provide a relation between the electrical radius of the waveguide (or frequency) and the phase constant of each mode. The phase constant is found from k and k_c as

$$\beta=\sqrt{k^2-k_c^2}. \tag{6}$$

For a waveguide with PEC walls, the transverse wavenumber, k_c, is independent of k — this is not generally true for impedance surface waveguides.

For the purposes of this analysis, we shall concentrate on impedances which satisfy the *balanced hybrid condition* and thus support balanced hybrid modes. These are modes for which $|\gamma|=1$. The balanced HE modes are desirable, as they approach totally parallel field lines when k and β are close. It can be shown that the balanced hybrid condition is fulfilled when

$$z_\phi=\frac{1}{z_z} \quad \text{or} \quad x_\phi=-\frac{1}{x_z}. \tag{7}$$

where x_z and x_ϕ are normalized reactances, i.e. for lossless surfaces $z_z = jx_z$ and $z_\phi = jx_\phi$. Under the balanced condition balanced EH modes are also present ($\gamma=-1$) in addition to the HE modes. These are usually not desirable and one would want to avoid exciting them.

In a classical matched feed design, the higher order mode is excited at the beginning of the horn and then propagates along with the primary mode in the rest of the horn. The phase relationship between the modes at the aperture of the horn is of vital importance. Thus, we want the phase constants of the two modes to have the same difference over the frequency band of interest, thereby making the phase difference at the aperture constant. This is impossible for smooth walled horns with the TE_{21} compensating mode.

Additionally, if the compensating mode is excited along a distributed section of the impedance waveguide, we know from directional coupler theory [11], that the two modes must have not only a constant difference in phase constant, but the *same* phase constant.

A. Soft surface

Fig. 1 shows the dispersion diagram of a waveguide with infinite longitudinal and zero azimuthal impedance — corresponding to a *soft* surface, e.g. a transversely corrugated surface at resonance. This surface satisfies the hybrid condition with both sides of Eq. (7) being zero. In a real corrugated waveguide, the surface impedance would change with frequency, resulting in different dispersion characteristics, but Fig. 1 shows results qualitatively similar to those at the aperture section of a corrugated horn. Evidently, the HE_{11} and HE_{21} modes have different dispersion characteristics. The two modes have varying difference in phase constant with frequency. Consequently, if they are allowed to propagate together for a length of waveguide, their mutual phase difference at the end of the section will change with frequency — which is exactly what should be avoided. This indicates that traditional corrugated horns have no advantage over smooth horns in the sense of matched feeds.

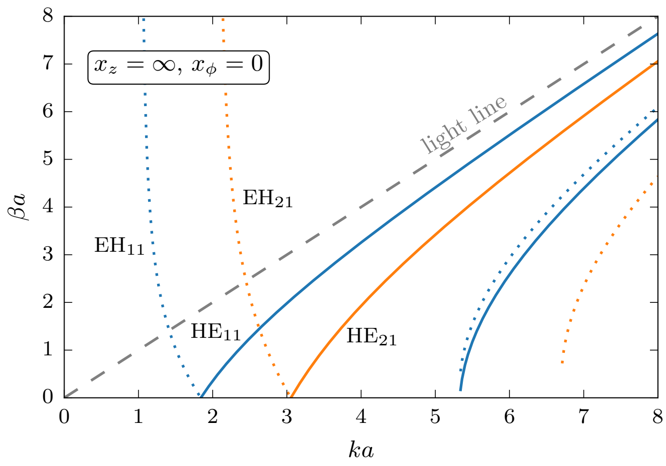

Fig. 1. Dispersion diagram for a circular waveguide with infinite longitudinal impedance, $x_z=\infty$ and zero azimuthal, $x_\phi=0$. This corresponds to a corrugated surface where the corrugations are $\lambda/4$ deep. Blue curves represent $m=1$ azimuthal variation and orange curves represent $m=2$ variation. Full lines are HE modes and dotted lines are EH modes.

B. Small Capacitive Longitudinal Impedance

It turns out that impedance pairs where the longitudinal reactance, x_z, is negative (capacitive) and small, are more interesting for our purposes. We still require the balanced hybrid condition, Eq. (7), to be met. Fig. 2 shows examples of surfaces in this range with x_z equal to -1, -0.5, and -0.1, respectively. For $x_z=-1$, the HE_{11} and HE_{21} modes are much closer than in the ”corrugated” case of Fig. 1, and also have a more constant difference. Decreasing negative values of x_z moves the two phase constants even closer, and closer to that of free space. A surface of this type would be highly desirable for matched feed applications

IV. Implementation

The surface impedances in the previous section are idealized. An optimized surface for the matched feed purposes of

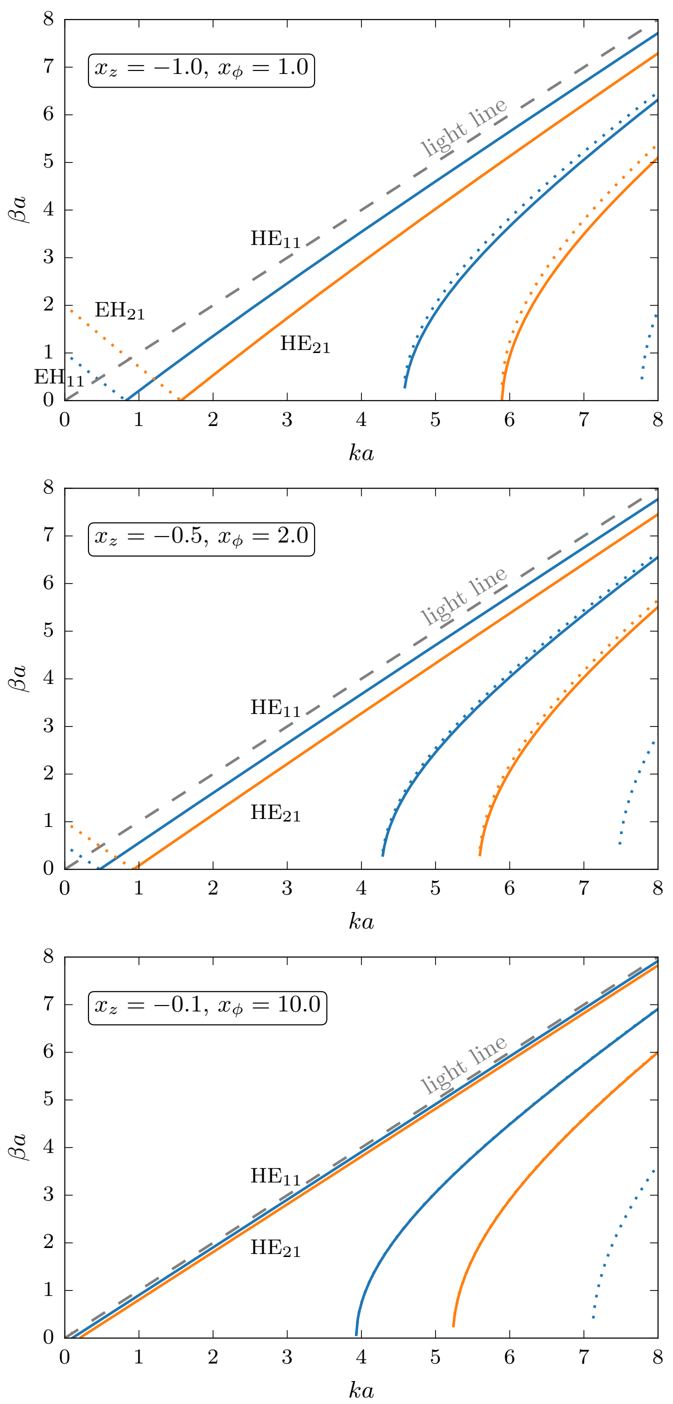

Fig. 2. Dispersion diagrams for a circular waveguide with three sets of surface reactances fulfilling the balanced hybrid condition: $x_z x_\phi = -1$. Blue curves represent $m = 1$ azimuthal variation and orange curves represent $m = 2$ variation. Full lines are HE modes and dotted lines are EH modes.

this paper, has not yet been found. However, looking in the literature, it does seem feasible. The horns designed by Wu et al. and Scarborough et al. [5], [6], [7], [8], feature impedances which are balanced and have a normalized longitudinal reactance of around $x_z = -1$. Again, their goal for optimization is not matched feeds, but the resulting surface happens to be promising for matched feed applications. The same is true for the surface designed in [12]. The surface designs might be even better suited if an optimization goal was added to possible make $|x_z|$ smaller.

V. Conclusion

We have made a study of circular cylindrical waveguides with anisotropic impedance walls for the possible application to offset reflector matched feed horns. The suitability of such waveguides relies on the dispersion characteristics of the primary HE_{11} mode and the HE_{21} mode used to compensate cross polarization. It is desirable that the phase constant of the two modes are close, or at least that the difference in phase constant does not change with frequency.

We can conclude that corrugated waveguides are unsuited to meet these requirements, but other balanced impedance surfaces are well suited. These surfaces are characterized by a capacitive (negative) reactance in the longitudinal direction and an inductive (positive) reactance in the azimuthal direction. Surfaces like this are shown in the literature to be realizable, and thus provide a promising platform for novel matched feed designs.

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